## ERRATA for Discrete Structures Book (last updated March 2016)

Note: These errata were found in the first printing of the book. They will be corrected in subsequent printings
NOTATION INDEX (inside front cover):
$\approx$ (symbol page 1; General, second symbol) Change 1.333 to 0.333
$\bigcap_{i \in I} A_{i}$ (last symbol on page 1) Change "for at every" to "for every"

## CHAPTER 1

Page 2: line - 2 (in footnote), change reference " $[\mathrm{Kli}-00]$ " to " $[\mathrm{Kle}-00]$ "
Page 4: line 7, change "most all" to "most"
Page 4: line -4 above Definition 1.4: change "is true of both" to "is true if both"
Page 9: footnote, line 4, change "Since a the" to "Since the"
Page 15: last line of footnote, change "proof by contraction" to "proof by contradiction"
Page 16: paragraph 4, line 1, change "present in two" to "present two"
Page 22: Exercise 22, change last sentence to: "From this information, is it possible to determine the bank
robber? Explain your answer."
Page 23: Exercise 36, line 3, replace "true in all other cases" to "false in all other cases"
Page 34: line 9, change "predicates" to "predicate"
Page 38: line 2 of "Solution", change "Although each" to "Each"
Page 38: line -2 of footnote, change "into" to "to"
Page 40, line 4, change "all of" to "of all"
Page 40, line -3 , change "are called infinite sets" to "is called an infinite set"
Page 41, Example 1.11, line 4, change "Joey" to "Joey (a male)"
Page 45: line 3, change "sets" to "sides"
Page 49: last line of Definition 1.18, insert "by" after first "denoted", replace "is denoted by" by "or"
Page 50: Paragraph 2, line 8, change "Egyptian" to "Greek"
Page 50: Paragraph 2, line -3, change "course in" to "course on"
Page 54: Exercise 10(c), change "not more" to "not always more"
Page 54: Exercise 12, part (b) Change " $(A \cup C) \cap(B \cup C)$ " to " $(A \cup B) \cap(A \cup C)$ "

## CHAPTER 2:

Page 62: paragraph 3, line 1, replace "is only" with "is used only"
Page 65: line -2, replace "elements of $S$ " with "elements of $A$ "
Page 66: line 2, replace "set of $T$ " by "set $T$ "
Page 66: paragraph 2, line 8, replace "one" by "on"
Page 66: Example 2.3, line 3, insert a colon ":" after "countries"
Page 70: line 3, change "from" to "form"
Page 74: Exercise 26, part (b), change " $f^{-1}\left(A_{1}\right) \sim f^{-1}\left(A_{2}\right)$ " to " $f\left(A_{1}\right) \sim f\left(A_{2}\right)$ "
Page 78: Definition 2.8, change " $m$ is a factor of $b-a$ " to " $m$ is a factor of $a-b$ "
Page 79: Paragraph 2, line 3, replace "move down" by "moving right through"
Page 80: Line before Proposition 2.2, change " 2.9 " to "2.8(a)"
Page 83: Exercise 3(d), change "attended the same high school" to "graduated from the same high school"
Page 85: last line, insert " 15 " between 12 and 20.
Page 88: Exercise 22(a), to the relation $R$ add the element " $(b, e)$ " (needed for transitivity)
Page 88: Exercise 22(c), add the element "4" to the set

## CHAPTER 3:

Page 111: Paragraph 3, line 6, change "algorithms" to "factoring algorithms"
Page 111: Last line before (Section) 3.1, change "Chapter 6" to "Chapter 4"
Page 113: Paragraph 1, line -2 , change " $\mathrm{S}(\mathrm{k}+1)$ " to " $\mathrm{S}(\mathrm{k}+1)$ )"
Page 117: Paragraph 3, line -2, change "expect" to "except"
Page 126: line 5 below figure caption, change "pegs" to "disks"
Page 127: line 2, change "largest peg" to "largest disk"

Page 131: Exercise for the Reader 3.8: Replace "fibonnacciRec" with "fibonnacciv2" in parts (b) and (c). Page 132: Definition 3.2, Reword as follows: If $N$ is an integer, and $k$ is a nonnegative integer, a definition of the form:

$$
\left\{\begin{array}{l}
\text { 1. (Basis Step) } a_{N}=\alpha_{N}, a_{N+1}=\alpha_{N+1}, \cdots a_{N+k-1}=\alpha_{N+k-1}, \\
\text { 2. (Recursive Step) For } n \geq N+k, a_{n}=f\left(a_{n-1}, a_{n-2}, \cdots, a_{n-k}, n\right),
\end{array}\right.
$$

where $f$ is some function of $k+1$ variables, will define an infinite sequence $\left\{a_{n}\right\}_{n=N}^{\infty}$. Such a sequence is said to be recursively defined, and to have degree $k$.
Page 132: line -2 or Paragraph -3, replace " $f\left(a_{n}, a_{n+1}, \cdots, a_{n+k}\right)$ " by " $f\left(a_{n-1}, a_{n-2}, \cdots, a_{n-k}, n\right)$ "
Page 137: In line 1 of Example 3.10 and line 3 of its solution change "Theorem 2.6" to "Theorem 3.6"
Page 139: Theorem 3.9, line 1: change "linea" to "linear"
Page 144: Exercise 21 should refer to 20 instead of 18.
Page 145: Exercise 29(c), change $\left\{\begin{array}{l}a_{1}=0, a_{2}=2 \\ a_{n}=a_{n-1}+a_{n-2}\end{array}(n \geq 3)\right.$ (o $\left\{\begin{array}{l}a_{0}=0, a_{1}=2 \\ a_{n}=a_{n-1}+a_{n-2}(n \geq 2)\end{array}\right.$
Page 155: Paragraph after Theorem, line 9, change "the preceding" to "a recent"
Page 155: Last Paragraph, line 3, change " 5 -digit" to " 50 -digit"
Page 158: Algorithm 3.1, displayed equations. The indices on the some of the q's in the displayed equations are off; change as follows:

$$
\begin{array}{rlrlr}
b= & q_{2} r_{1}+r_{2}, & 0 \leq r_{2}<r_{1} & & \\
r_{1}= & q_{2} r_{2}+r_{3}, & 0 \leq r_{3}<r_{2} & & \rightarrow r_{1}=q_{3} r_{2}+r_{3} \\
& \ldots & & \\
r_{n-2}= & q_{n-1} r_{n-1}+r_{n}, & 0 \leq r_{n}<r_{n-1} & \rightarrow r_{n-2}=q_{n} r_{n-1}+r_{n} \\
r_{n-1}= & q_{n} r_{n}+0 & & \rightarrow r_{n-1}=q_{n+1} r_{n}+0
\end{array}
$$

The corresponding changes in the proof of Theorem 3.16 on page 159 should be made: Basically, each instance of the form $r_{i-1}=q_{i} r_{i}+r_{i+1}$ should be changed to $r_{i-1}=q_{i+1} r_{i}+r_{i+1}$
Page 161: line 2 of Solution, change "fraction" to "quotient"
Page 162: Theorem 3.18, line 2, change " $m$ " to " $m>1$ "
Page 167: line -2 , change "mod 24 " to "mod 58 "
Page 168: line 1 of Example 3.19, insert "mod $n$ " after the word "orders"
Page 169: line 2 of Definition 3.10, insert "mod $n$ " after "of $g$ "
Page 172: Exercises 15 and 16 should refer to Exercises 13 and 14, resp., rather than Exercises 11 and 12.
Page 175: line 5, change "prime for" to "prime"

## CHAPTER 4

Page 189: $2^{\text {nd }}$ line after "Solution:" paragraph. Change " $128+32+16+8+2+1=187$ " to " $128+16+8+2+1=$ 155"
Page 192: lines 2 and 3 after Table, change both instances of " 100001 " to " 10001 "
Page 215: lines 2, 8, 10, delete the redundant periods (there is one on each line)

## CHAPTER 5

Page 315: line 2, second factor of 99 should be changed to 98
Page 318: Solution to Example 5.5, third and fourth-to-last lines, change both " $A \cap P \cap \sim E$ " and " $A \cap \sim P \cap E$ " to " $A \cap P \cap E$ "
Page 321: Figure 5.5, change the angle label from " $\pi / 6$ " to " $\pi / 3$ "
Page 321: line 2, insert "congruent" before "arcs"
Page 321: line 8, change "the" to "its"
Page 321: line -3 , change " 367 " to " 366 ", line -2 , change " 1101 " to " 1097 "
Page 329: line 2 of Definition 5.2, change "(1)" to "(2)"
Page 331: last line of part (c), change " $1,378,820$ " to " $1,378,830$ "
Page 333: Part (b): Revise as follows: We use the complement principle (often useful for sets described using the phrase "at least"). The number of poker hands with at least one pair is the total number of poker hands less the number that contain no pairs. The number with no pairs can be easily counted using the multiplication principle:

There are $\mathrm{C}(13,5)$ ways to choose 5 different denominations in the hand and for each denomination we have four choices for the suit. So the answer is $C(52,5)-C(13,5) \cdot 4^{5}=1,281,072$.
Page 334: line 2 of NOTATION, change " $k$ objects taken $n$ at a time" to " $n$ objects taken $k$ at a time"
Page 335: line 1 of SOLUTION, change "(5)" to "(6)"
Page 337: last line, delete the second instance of "that"
Page 338: line 7 of Proof 1, change "correspond" to "corresponds"
Page 344: line 4, change "amd" to "and"
Page 346: Exercise 39: Change the second factor " $\binom{n}{r-k}$ " to "" $\binom{m}{r-k}$ "
Page 359: second-to-last paragraph, line 6, change "first" to "second"

## CHAPTER 6

Page 395: line 5, change "test positive for" to "actually have"
Page 405: line 1, change "at least one" to "at least one 1"
Page 431: line -2 , replace " $=\mathrm{E}[X]^{2}-2 \mu \mathrm{E}[X]+\mu^{2} \mathrm{E}[1]=\mathrm{E}[X]^{2}$ " with " $=\mathrm{E}\left[X^{2}\right]-2 \mu \mathrm{E}[X]+\mu^{2} \mathrm{E}[1]=\mathrm{E}\left[X^{2}\right]$ "

## CHAPTER 7

Page 474: line 9, replace all after the word "kept..." with "made it standard in both the mathematical and computer science literature.

## CHAPTER 8:

Page 509: Example 8.6, Part (b), line 3, delete " $C_{1}, C_{2}$," from the list " $C_{1}, C_{2}, C_{3}$ (etc)"
Page 512: Figure 8.17, modify the label on the left graph from " $Q_{3}$ " to " $G=Q_{3}$ ".
Page 516: EXERCISE FOR THE READER 8.12, change matrix from " $A=\left[\begin{array}{llll}0 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 4 \\ 3 & 3 & 4 & 3\end{array}\right]$ " to " $A=\left[\begin{array}{llll}0 & 1 & 3 & 2 \\ 1 & 1 & 3 & 0 \\ 3 & 3 & 3 & 4 \\ 2 & 0 & 4 & 2\end{array}\right]$ "
(so that answer in the back will match).
Page 520: Figure 8.25, in the caption, line 2, change "with two parallel edges and one self-loop, with edge labels" to "with a self loop and with two nonparallel edges between vertices $u$ and $v$ "
Page 525: Exercise 5(c), change " $(2,2,1,1,1,1)$ " to " $(2,1,1,1,1)$ "
Page 526: Exercise 11(c), change "( $5,5,4,4$ )" to " $(5,5,4,2)$ "
Page 526: Figure 8.31, in the caption change "Four simple graphs" to "Four general graphs"
Page 526: Exercise 6(a), change sequence to " $(4,3,3,3,1)$ "-intention was to make it graphic
Page 529: The h-label on K should be " $h_{3}$ " rather than " $h_{1}$ "
Page 541: last line (before footnote), change " $P=<a, b, h, a, \ldots$ " to " $P=<a, b, h, g, \ldots$ "
Page 546: line 4 of Proposition 8.5, change " $w=v_{n}{ }^{\prime}$ " to " $w=v_{m}{ }^{\prime}$ "
Page 556: last line: change " $Y=\{y \in V: \operatorname{dist}(u, y)$ is even $\}$ " to " $Y=\{y \in V: \operatorname{dist}(u, y)$ is odd $\}$ "
Page 557: above Exercises 8.2, line -6: change "areboth" to "are both"
Page 560: Exercise 15(b), change "row 1 column 2" to "row 2 column 3"
Exercise 15(e) , change "row 2" to "row 4"
Page 562: line 5, replace "Part (a)" with "Part (c)"
Page 564: \#37, Suggestion: change "61(d)" to "62(b)"
Page 574: line 5, change "Proposition 8.9(a)" (which is still correct) to "the hypothesis" (which is better), also change "tree" to "graph"
Page 577: line 1 of paragraph that begins with "Typically,", change "the a given" to "a given"
Page 580: Paragraph 3, line 2: change " $0 \leq k<h$ " to " $0 \leq k \leq h$ "
Page 582: Paragraph 2, change "Another alternative" to "this alternative".

Page 582: OBSERVATION: The sentence should read: In any full $m$-ary tree, $\ell=(m-1) i+1$, where $\ell$ is the number of leaves and $i$ is the number of internal vertices. The proof should read: Since $n=\ell+i$, formula (4) of Theorem $8.13 n=m i+1$ implies that $\ell+i=m i+1 \Rightarrow \ell=(m-1) i+1$.
Page 596: Exercise 7, second "(a)" should be "(b)"
Page 607: Equation (5), change " $u_{1}$ " to " $u_{i}$ "

## CHAPTER 10:

Page 764: In line 5 and inTheorem 10.1, change " $R(3,3)=9$ " to " $R(3,3)=6$ " (this is a simple typo; the correct result is discussed and proved)

## APPENDIX B:

Page 789: EFR 1.10 second-to-last Line, change "by definition of universal quantification" to "by definition of existential quantification"
Page 790: EFR 2.3; solutions of parts (b) and (c) are missing; here they are as follows:
(b) This familiar function is neither one-to-one (e.g., since $f(-1)=f(1)$ ) nor onto (since $f(\mathbb{R})=[0, \infty) \neq \mathbb{R}$ ).
(c) $F$ has the same formula as the function in part (c), but its domain has been cut down to $[0, \infty$ ), so it satisfies the horizontal line test and thus it one-to-one. Also, the co-domain has been cut down to the range $F(\mathbb{R})=[0, \infty)$, so $F$ is onto.
Page 793: EFR 2.12 (b), insert " 15 " in the set A, and in the diagram put the point for 15 on the central line, between 6 and 20 with lines going from it to 3,5 , and 30 .
Page 803: EFR 3.21, line -2 of Method 1, change " 8052 " to " 802 "
Page 814: EFR 5.2, line 5, change " $D$ " to " $D$ _ 2 "
Page 825: EFR 6.9, in line 2, change " $3 / 3$ " to " $3 / 8$, in line -2 , change " $(3 / 8)$ " to " $(3 / 4)$ "
Page 830: EFR 6.18, line -7 , set first probability " $=0$ "
Page 831: last line, replace "( $n-k$ )" in the first sum by "( $n-1-k$ )"
Page 839: EFR 8.8, in the last line, change 0.012 to 0.0012 .
Page 842: EFR 8.21, first line after matrices, change "Algorithm 9.2" to "Algorithm 8.2"
Page 844: EFR 8.31, line - 2 of Part (b), change "left children" to "right children"

## APPENDIX C:

Page 859: item 11(b), change last shaded truth value from " $T$ " to " $F$ " (in truth table)
Page 863: Sec. 1.2, item 5(e), left parenthesis missing in logical formula, change "... $x \neq y$ )..." to "...( $x \neq y) \ldots$
Page 864: first paragraph, item 9(e), in the logical formulas change all three instances of " $\sim$ " to " $G$ ", all two instances of " $l$ " to " $G^{\prime}$ ", and in the final English sentence, change "...no one listens.." to "... no one gossips..." Page 864: item 17(e)(ii), change " $b=1$ " to " $b=0$ "
Page 866: item 15(b), (for clarity) add the following to the beginning for the proof: "Assume that $A \Delta B=B \Delta C$.
We need to show that $A=B$. We will do this by proving that $A \sim B=\varnothing$ and $B \sim A=\varnothing$."
Page 868: figure at bottom left of page, change caption from "Figure for \#3(b)" to "Figure for \#3(a)"
Page 869: item 11(e), change " $\pi / 2+2 n \pi$ " to " $2 n \pi$ "
Page 869: item 13(b), change " $\mathbb{R}_{+}=[0, \infty)$ " to " $\mathbb{R}_{+}=(0, \infty)$ "
Page 872: last line, change " $f(2)$ must be 2,3 , or 4 " to " $f(2)$ must be 2 or 4 "
Page 873: first line, change " $f(2)$ must be 3 or 4 " to " $f(2)$ must be 4 "
Page 873: item label \#23 should have been labelled \#33
Page 883: line 1, change " $(4 k+1)$ " to " $(4 k-1)$ "
Page 883: item 9, line -3 , append " $(k+1)([k+1]+1)$ " to end of line
Page 883: item 9, line -1 , change " $(k+1)(k+2) / 3\{k+3\}$ " to " $[(k+1)(k+2) / 3]\{k+3\}$ " (for added clarity)
Page 890: item 29(c), line 3, change " $a_{1}=0, a_{2}=2$ " to " $a_{0}=0, a_{1}=2$ "
Page 907: item 11(b), change "... = 216" to "... = 108"
Page 907: item 13(b)(c), answer given to (b) is really the answer to (c); the correct answer to (b) is
$\sum_{j=5}^{7}\left[62^{j}-\left(52^{j}+10^{j}\right)\right]=2,531,097,358,400$
Page 907: item 13(d), answer should be: $3 \cdot 8 \cdot 62^{2}+\left[4 \cdot 8 \cdot 62^{3}-8^{2}\right]+\left[5 \cdot 8 \cdot 62^{4}-3 \cdot 8^{2} \cdot 62\right]=598,760,224$

Page 907: item 15(b), change " 5766 " to " 3116 ", and in (c)/last sentence, add "and the three " $==$ " to " $==$ " Page 908: line 2, replace "pigeons" with "pigeonholes"
Page 908: item 25(a) change to: $16 x^{4}-64 x^{3} y+160 x^{3} z+96 x^{2} y^{2}-480 x^{2} y z+600 x^{2} z^{2}-64 x y^{3}+480 x y^{2} z$ $-1200 x y z^{2}+1000 x z^{3}+16 y^{4}-160 y^{3} z+600 y^{2} z^{2}-1000 y z^{3}+625 z^{4}$ (original was answer to 24(a))
Page 913: item 1 (a), change " $1 / 2^{\prime \prime}$ to " $2 / 3$ ", item 1 (c), change answer to: $1-1 / 16-4 / 16=11 / 16 ; 1-1 / 2^{7}$ $-7 / 2^{7}=15 / 16$
Page 909: line -5, change "Theorem 3.5" to "Proposition 3.5"
Page 910: item 5(b), change final answer from "exp(2x)" to "exp(-2x)"
Page 910: line 2 of item $7(a)$, change " $(n=0,1,2,3,4)$ " to "if $n=3,4,5,6,7$, and $a \_n=0$ if $n=0,1,2$ "
Page 910: line 3 of item $7(b)$, change "a_1 = 3 " to "a_1 = -3 " and after "a_3 = -1 ", insert "a_4 = 0 ."
Page 910: item $7(\mathrm{f})$ : solution given is actually for Exercise 8(f), replace with the following:
Using (9), we may write: $e^{2 x}\left(1-x^{2}\right)=\sum_{n=0}^{\infty}\left[x^{n} / n!\right]\left(1-x^{2}\right)=\sum_{n=0}^{\infty}\left[2^{n} / n!\right] x^{n}-\sum_{n=0}^{\infty}\left[2^{n} / n!\right] x^{n+2}=\sum_{n=0}^{\infty}\left[2^{n} / n!\right] x^{n}-\sum_{n=2}^{\infty}\left[2^{n-2} /(n-2)!\right] x^{n}$,
so the sequence is $\left.a_{0}=1, a_{1}=2, a_{n}=2^{n} / n!\right]-2^{n-2} /(n-2)!(n=2,3,4, \cdots)$
Page 910: item 13(b), change answer to " -0.02734375 "
Page 910: item 15(a), change " $(1+x)^{\wedge} 1 / 2^{\prime \prime}$ to " $(1+x)^{\wedge}(-1 / 2)^{\prime}$
Page 921: item1(e): solution given is actually for Exercise 2(e), replace with the following:
True: The big-O relationship can be rewritten as: $n^{2}=O\left(2^{n}\right)$.
(i) Claim: For $n \geq 4, n^{2} \leq 2^{n}$. Proof of Claim by induction: Basis step: $n=4$, true since both sides equal 16 .

Inductive step: Assume the inequality is true for an integer $k \geq 4$, i.e., $k^{2} \leq 2^{k}$. We must show that it is also true for the integer $k+1$, i.e, that $(k+1)^{2} \leq 2^{k+1}$. Indeed, using the inductive hypothesis, we obtain that:

$$
(k+1)^{2} \leq k^{2}+(2 k+1) \leq 2^{k}+3 k<2^{k}+k^{2} \leq 2^{k}+2^{k}=2^{k+1},
$$

which completes the inductive step and hence the proof of the claim. The Claim shows us that $k=4$ and $C=1$ are witnesses to the asserted big-O relationship.
(ii) The big-O relationship is a direct consequence of Proposition 7.1(c).

Page 924: item 3(c), change " $\operatorname{deg}_{G_{1}}(u)=2 "$ to $" \operatorname{deg}_{G_{1}}(u)=3 "$
Page 929: item (iv), in the matrix $B\left(G_{4}\right)$, change the fourth row (labelled by "x") to: 001101101101
Page 929: item 33(a), switch the sets for V(G_3) and V(G_4)
Page 935: item 25(b), change " 3 " to " $\min \{n, m\}$ "
Page 938: item 7(b), change "five" to "six"
Page 939: item 13(b), has two instances labelled the same, the second label should be "(c)"
Page 939: item 15(a), change " $\mathrm{h}=5$ " to " $\mathrm{h}=4$ " and change "five" to "four"
Page 942: line -6, before "We" insert the following:
If $\mathrm{n}=0$, the equation becomes $d_{1}=0$, while if $\mathrm{n}=1$, it becomes $d_{1}+d_{2}=2$. These two cases are easily realized by the trees $K_{1}$ and $K_{2}$, respectively.

## REFERENCES:

Page 959: add this reference: [HoSh-93] Holton, D. A., and J. Sheehan, The Petersen Graph, Cambridge University Press, Cambridge UK (1993)

